

الامتحان مكون من (5) أسئلة مكتوبة في صفحة واحدة و المطلوب الإجابة على كل الأسئلة 0

Time: 3 Hours

(1)(a) Test the series: $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{n^2}$

(b) Show that the envelope of the curves: $(x - \cos \alpha)^2 + (y - \sin \alpha)^2 = 1$
 is the circle: $x^2 + y^2 = 4$

(c) Find the extrema of the function: $f(x, y) = x^3 + \frac{1}{2}x^2y - 2x^2 - \frac{1}{4}y^2$

(2)(a) Find the interval of convergence of the series: $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{2n+1}$

(b) Verify Euler's theorem for the function $f(x, y, z) = x^2y + xy^2 + z^3$

(c) Solve the equation: $(\cos x + y \sin x)dx + (4y - \cos x)dy = 0$

(3) Solve the differential equations:

(a) $y' + \frac{2}{x+1}y = x^3$

(b) $y'' + y = (1 + 2 \cos x)^2$

(c) $(x^2 D^2 - xD + 1)y = x^4 + 2$

(4)(a) Solve the equation: $y'' + y = \sec x$

(b) If $\phi = xyz$ and $\bar{U} = 2xi + yzj + xzk$. Find $\nabla \cdot (\phi \bar{U})$ and $\nabla \times (\phi \bar{U})$

(c) Evaluate the integral: $\int_0^1 \int_y^1 6y e^{x^3} dx dy$

(5)(a) Find the flux of the vector field: $\bar{U} = 2xi + (y + z)j + xyk$

through the surface $x^2 + y^2 + z = 1, z \geq 0$

(b) Verify Green's theorem for the integral: $\oint_C (1 - xy)dx + (xy + 3)dy$

where C is the circle $x^2 + y^2 = 1$

Good Luck

Dr. Mohamed H. Eid

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Time: 3 Hours

(1)(a) Test the series: (i) $\sum_{n=1}^{\infty} \frac{2^n}{2n+1}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+1}$

(b) Find the interval of convergence of the series: $\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n+2}}$

(c) Find the envelope of the curves: $(y+\alpha)^2 + (x-\alpha)^2 = 2$

(2)(a) Find the extrema of the function: $f(x,y) = x^2 + y^3 - 4xy + 4y$

(b) Solve the differential equation: $y'' + y = \tan x$

(3) Solve the differential equations:

(a) $(x^3 + \cos y)dy + (x + 3y x^2)dx = 0$

(b) $y'' + y = e^{2x} + \cos x$

(c) $x^2 y'' - 3xy' - 5y = x^2$

(4)(a) Solve the system of equations: $(D+1)y - z = x,$

$-2Dy + (D+1)z = e^{2x}$

(b) If $\bar{U} = (2xy)i + (xy z^2)j - (xz y^2)k$. Find $\nabla \cdot \bar{U}$ and $\nabla \times \bar{U}$

(5)(a) Show that the function $u(x,y) = y + e^{2x} \cos 2y$ is harmonic and find its conjugate $v(x,y)$ such that the function $w = u + iv$ is analytic

(b) Evaluate the integrals: (i) $\int_C \frac{z \cos 3z}{z^2 + 36} dz$ (ii) $\int_C \frac{\sin 2z}{z^3} dz$

where C (in the two integrals) is the ellipse $|z-4| + |z+4| = 10$

Good Luck

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